

In this talk, I will show how we can obtain sharp upper bounds on the length of the shortest closed geodesic on a punctured sphere with three or four ends endowed with a complete Riemannian metric of finite area using ramified covers from the torus to the sphere. In both cases, I will describe the extremal metrics, which are modeled on the Calabi-Croke sphere and the tetrahedral sphere. Note that the Calabi-Croke sphere is obtained by gluing two copies of an equilateral triangles along their boundaries, and the tetrahedral sphere is given by the regular tetrahedron.