I will present some outcomes of the theory of convergence for geometric actions of groups G on CAT(0)-spaces X and their quotients M=G\X ("CAT(0)-orbispaces"), a joint research with Nicola Cavallucci (arXiv:2109.13025, arXiv:2210.01085, arXiv:2304.10763).

We are interested in the equivariant, Gromov-Hausdorff convergence of actions in the sense of Fukaya, and we restrict our study to geometric actions of groups on CAT(0)-spaces X which are geodesically complete and P-packed at some scale R: this is a metric condition which replaces, at macroscopic scale, a lower bound of the scalar curvature and provides many useful tools for CAT(0)-spaces (an analogue of the classical Margulis' Lemma, a uniform control of the measure of small balls, compactness in pointed Gromov-Hausdorff topology etc).

In the talk, I will describe some splitting, finiteness, compactness and isolation results for uniformly cobounded geometric actions on uniformly packed CAT(0)-spaces, which stem from a careful investigation of how collapsing can occur in this class of geometric actions.

For instance, we prove that :

- there exist only finitely many groups G acting geometrically and non-singularly on (P,R)-packed CAT(0)-spaces with co-diameter less than D;
- the class of (P,R)-packed, CAT(0)-homology orbifolds M=G\X with diameter at most D is compact;
- flats are isolated in the above class.

These results generalize (and, in some cases, improve) some classical theorems for nonpositively curved Riemannian manifolds.